Math 54
Faculty: Professor Nadler/Paulin
Study Group Leader: Trevor Wu
(trevorhwu@berkeley.edu)
Location: SLC

Math 54: Diagonalization

Review: Eigenswag and Bases

1. If a matrix **A**, a non-zero -vector **x**, and a constant **λ**, satisfy the equation Ax = λ, then λ is the _______________ and x is the _______________.
2. Write the vector $\begin{bmatrix}1\\1\end{bmatrix}$ in its B − coordinates of basis B = $\left\{\begin{bmatrix}0\\-1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}\right\}$:

Guided Introduction

$$A = \begin{bmatrix}1 & 4\\0 & 3\end{bmatrix}$$

3. Consider this matrix A
   a. Find its eigenvalues and corresponding eigenvectors

   b. Create a matrix from the bases of the eigenspaces

   c. Find the change of bases matrix from the standard basis to the eigenspace

   d. If T(x) is a transformation = Ax for an x in R^2, then what would $[T]_B$ for the basis previously found?

      i. What do you notice about $[T]_B$?

Definitions

Diagonalization is a way of factoring a matrix A from its eigenvalues and eigenvectors into the form $A = PDP^{-1}$. P is the matrix of the basis of all eigenvectors, while D is a diagonal matrix of the corresponding eigenvalues.

Two matrices A and B are **similar** if A can be written as PBP^{-1}.

An nxn matrix is only diagonalizable if it has n **distinct eigenvalues** (including multiplicities), in other words, if its eigenvectors span R^n.

Math 54
Faculty: Professor Nadler/Paulin
Study Group Leader: Trevor Wu
(trevorhwu@berkeley.edu)
Location: SLC

Steps for Diagonalization

1. Find Eigenvalues and Eigenvectors of matrix, here, you should find out whether the matrix is diagonalizable.
2. Construct a matrix P from the eigenspaces for each eigenvector.
3. Find P^{-1}
4. Construct a diagonal matrix D from the eigenvalues, make sure each eigenvalue matches to its specific eigenvector in P.

Practice Diagonalization

1. Find the diagonalized representation of A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

2. How would you find A^{100} with diagonalization if $A = \begin{bmatrix} 1 & 4 \\ 0 & 3 \end{bmatrix}$.

3. Book Problem: Find the diagonalized representation of A = $\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

Foreshadowing

1. Spectral Theorem:
   Find the eigenvectors of the symmetric matrix A = $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$. Then find the dot product of the eigenvectors.
   a. What do you notice about the eigenvectors?
   b. How can you write the matrix in the form $A = PDP^T$?

2. Singular Value Decomposition:
   It turns out you can "diagonalize" matrices that are not square?
   a. How do you think this can be done?
   b. What would be the "eigenvalues"?